**Bridging the gap between GCSE and AS/A Level Mathematics**

The object of these exercises is to help you get started with the A Level course, and to smooth your path through it. Many students find A Level a challenge compared with GCSE. This is a recognised issue – if you find this you are very far from being alone! I hope that these pages will help you.

The main focus is on developing skills, as opposed to learning new material. So I suggest that you don’t approach it with the mind-set “what do I have to do to get full marks?” but “what can I learn that will help me in my future studies?” You want to be fluent in a number of aspects of GCSE work – not just able to get an answer that would score a mark in a GCSE examination. For example, you will want to get a result in a form that you yourself can go on and use readily. So there will be an emphasis on real fluency in algebra. Likewise, you should try to develop the ability to think of the shapes of graphs corresponding to algebraic formulae. Do not be afraid of any of this; over the years the vast majority of A Level students have succeeded with the course. But you will find the path easier, enjoy it more and be more confident if you are really on top of the basic vocabulary of the subject before you start putting it together into A Level sentences.

One recurring theme here is; just because you *can* multiply out brackets, it doesn’t mean that you *should*. Indeed, at this level it is often better to keep an expression in its factorised (bracketed) form.

Good luck!

**This booklet is only a collection of exercises – for full notes and examples on these topics (as well as the answers!) go to**

[**https://mathematics.bourne-grammar.lincs.sch.uk/**](https://mathematics.bourne-grammar.lincs.sch.uk/)

**Further reading**

There are not many books designed for the sort of transition that this booklet represents, but an outstanding exception is:

Fyfe, M. T., Jobbings, A. and Kilday, K. (2007) *Progress to Higher Mathematics*, Arbelos.

ISBN 9780955547706.

Alternatively, also published by Arbelos is an expansion of the same book which is more specifically aimed at the transition to A Level. You may not need quite so much as this:

Fyfe, M. T., and Kilday, K. (2011) *Progress to Advanced Mathematics*, Arbelos.

ISBN 9780955547737.

**Exercise 1.1 Simple Algebraic Expressions**

**1** Find the values of the letters *p*, *q* and *r* that make the following pairs of expressions always equal.

(a)  (b)  (c) 

**2** Solve the following equations.

(a)  (b)  (c) 

**3** Make cos *C* the subject of the formula *c*2 = *a*2 + *b*2 – 2*ab* cos *C*.

**4** (a)Multiply  by 8. (b) Multiply (*x* + 2) ÷ 3 by 12.

(c) Multiply by 6. (d) Multiplyby 8.

**5** Solve the following equations.

(a) **** (b) 

(c) 

**6** Make *x* the subject of the following equations.

(a)  (b) 

**Exercise 1.2 Algebraic Fractions**

**1** Work out the following. Answers may be left as improper fractions.

(a)  (b) 

(c)  (d) 

(e)  (f) 

(g)  (h) 

**2** Make *x* the subject of the following formulae.

(a) *A* = π*x*2 (b)  (c) (*u* + *v*) = *tx* (d) 

**3** Simplify the following compound fractions.

(a)  (b)  (c) 

**4** Write as single fractions.

(a)  (b)  (c) (d)  (e)  (f)  (g) 

**5** Write as single fractions.

(a)  (b)  (c) 

**Exercise 1.3 Quadratic Expressions**

**1** Write without brackets.

(a) (*x* + 5)2 (b) (*x* – 4)2 (c) (2*x* + 1)2

(d) (3*x* – 2)2 (e) (*x* + 2)(*x* – 2) (f) (3*x* + 4)(3*x* – 4)

**2** Simplify the following equations into the form *ax* + *by* + *c* = 0.

(a) (*x* + 3)2 + (*y* + 4)2 = (*x* – 2)2 + (*y* – 1)2

(b) (*x* + 5)2 + (*y* + 2)2 = (*x* – 5)2 + (*y* – 2)2

(c) (2*x* + 1)2 + (*y* – 3)2 = (2*x* + 3)2 + (*y* + 1)2

**3** Simplify the following where possible.

(a)  (b)  (c) 

(d)  (e)  (f) 

**4** Write the following in the form (*x* + *a*)2 + *b*.

(a) *x*2 + 8*x* + 19 (b) *x*2 – 10*x* + 23 (c) *x*2 + 2*x* – 4

(d) *x*2 – 4*x* – 3 (e) *x*2 – 3*x* + 2 (f) *x*2 – 5*x* – 6

**5** Write the following in the form *a*(*x* + *b*)2 + *c*.

(a) 3*x*2 + 6*x* + 7 (b) 5*x*2 – 20*x* + 17 (c) 2*x*2 + 10*x* + 13

**6** Write the following in the form (*ax* + *b*)2 + *c*.

(a) 4*x*2 + 12*x* + 14 (b) 9*x*2 – 12*x* – 1 (c) 16*x*2 + 40*x* + 22

**7** Factorise as fully as possible.

(a) *x*2 – 25 (b) 4*x*2 – 36 (c) 4*x*2 – 9*y*4

(d) 3*x*2 – 7*x* + 2 (e) 3*x*2 – 5*x* + 2 (f) 6*x*2 – 5*x* – 6

(g) 8*x*2 – 2*x* – 15

**8** Multiply out and simplify.

(a)  (b)  (c) 

**Exercise 1.4 Cancelling**

**1** Simplify the following as far as possible.

(a) 5*x* + 3*y* + 7*x* – 3*y* (b) 3*x*2 + 4*xy* + *y*2 + *x*2 – 4*xy* – *y*2.

(c)  (d) 

(e)  (f) 

(g)  (h) 

(i)  (j) 

**2** Make *x* the subject of the following formulae.

(a)  (b) 

**3** Simplify the following.

(a)  (b) 

4 Simplify as far as possible.

(a)  (b) 

(c)  (d) 

(e) 

**Exercise 1.5 Simultaneous Equations**

Solve the following simultaneous equations.

**1** *x*2 + *xy* = 12 **2** *x*2 – 4*x* + *y*2 = 21

3*x* + *y* = 10 *y* = 3*x* – 21

**3** *x*2 + *xy* + *y*2 = 1 **4** *x*2 – 2*xy* + *y*2 = 1

*x* + 2*y* = –1 *y* = 2*x*

**Exercise 1.6 Fractional and negative powers, surds**

**1** Write the following as powers of *x*.

(a)  (b)  (c)  (d)  (e)  (f) 

**2** Write the following without negative or fractional powers.

(a) *x*–4 (b) *x*0 (c) *x*1/6 (d) *x*3/4  (e) *x*–3/2

**3** Write the following in the form *axn*.

(a)  (b)  (c)  (d)  (e) 6

**4** Write as sums of powers of *x*.

(a)  (b)  (c) 

**5** Write the following in surd form.

(a)  (b)  (c)  (d)  (e) 

**6** Rationalise the denominators in the following expressions.

(a)  (b)  (c) 

(d)  (e) 

**Trigonometry**

**1** Solve the following equations for 0 ≤ *x* < 360. Give your answers to the nearest 0.1°.

(a) sin *x*°= 0.9 (b) cos *x*°= 0.6 (c) tan *x*°= 2

(d) sin *x*°= –0.4 (e) cos *x*° = –0.5 (f) tan *x*° = –3

**2** Solve the following equations for –180 ≤ *x* < 180. Give your answers to the nearest 0.1°.

(a) sin *x*°= 0.9 (b) cos *x*°= 0.6 (c) tan *x*°= 2

(d) sin *x*°= –0.4 (e) cos *x*° = –0.5 (f) tan *x*°= –3

**3.** Find expressions, of the form *a* sin *θ* or *b* cos *θ*, for the sides labelled with letters in these triangles.

32°

*s*

*r*

5.6 cm

(a) (b)

20 cm

26°

*q*

*p*

e.g.

10 cm

17°

*u*

*t*

(c) (d)

8.4 cm

20°

*v*

*w*

**Exercise 3.1 Straight line graphs**

1 Rearrange the following in the form *y* = *mx* + *c*. Hence find the gradient and the *y*-intercept of each line.

(a) 2*x* + *y* = 8 (b) 4*x* – *y* + 9 = 0

(c) *x* + 5*y* = 10 (d) *x* – 3*y* = 15

(e) 2*x* + 3*y* + 12 = 0 (f) 5*x* – 2*y* = 20

(g) 3*x* + 5*y* = 17 (h) 7*x* – 4*y* + 18 = 0

**2** Sketch the following lines. Show on your sketches the coordinates of the intercepts of each line with the *x*-axis and with the *y*-axis.

(a) 2*x* + *y* = 8 (b) *x* + 5*y* = 10

(c) 2*x* + 3*y* = 12 (d) 3*x* + 5*y* = 30

(e) 3*x* – 2*y* = 12 (f) 4*x* + 5*y* + 20 = 0

**Exercise 3.2 Basic shapes of curved graphs**

Sketch (do not *plot*) the general shape of the graphs of the following curves. There is no need to find the intercepts, just identify the general shape.

Axes are not required but can be included in the questions marked with an asterix.

**1** *y* = *x*2 – 3*x* + 2 **2** *y* = –*x*2 + 5*x* + 1

**3** *y* = 1 – *x*2 **4** *y* = (*x* – 2)(*x* + 4)

**5** *y* = (3 – *x*)(2 + *x*) **6** *y* = (1 – *x*)(5 – *x*)

**7** *y* = *x*3 **8** *y* = –*x*3

**9\***  **10\*** 

**11** *y* = (*x* – 2)(*x* – 3)(*x* + 1) **12\*** 

**13** Sketch on the same axes the general shape of the graphs of *y* = *x*2 and *y* = *x*4.

**14** Sketch on the same axes the general shape of the graphs of *y* = *x*3 and *y* = *x*5.